

11. ON THE USEFULNESS OF THE KRIPKE-PRINCIPLE IN APPLIED DEONTIC LOGIC

Wojciech Żelaniec

CONTENTS: 1. Introduction – 2. Are there internally conditional obligations? – 3. Can we really distribute the “O” operator? – 4. Why the Kripke semantics leads us astray – 5. A modest proposal of an entirely this-worldly semantics and ontology for deontic propositions – 6. Lastly: How can the K-principle be wrong if in von Wright it is a tautology? – References.

1. INTRODUCTION

This is an essay in the philosophy of deontic logic, or rather in the philosophy that should – but in my view too seldom does – underlie deontic logic, not, as should be borne in mind all along, in deontic logic itself.

A purebred deontic logician will probably not at all take kindly to the question¹ whether deontic logic be any useful for disciplining “real-life” deontic reasoning. How presumptuous to ask questions like that! ... This said, if you are not, or not just, a purebred deontic logician, you may safely read on, without the risk of getting too much upset ... Now one moot point amongst many others concerning the usefulness of (most kinds of standard²) deontic logic for real-life deontic reasoning³ is the applicability of the so-called distributivity axiom, or K⁴-principle, which says, formally, that $O(A \rightarrow B) \rightarrow (OA \rightarrow OB)$ ⁵. In the version in which I shall here be subjecting to scrutiny it says, informally, this much: If (for any agent x)

¹ Posed, e.g., by C. Perelman, see C. Perelman 1968 or C. Perelman 1970.

² See, for an exposition, P. McNamara 2014.

³ Please note that the main thrust of this essay is not “deontic logic is bound to be useless for real-life deontic discourse, so let’s disregard it” but, much rather “let’s make deontic logic more useful for real-life deontic discourse”.

⁴ Short for “(Saul) Kripke”.

⁵ G.H. von Wright 1951, p. 13. Von Wright thinks (p. 5) that “it is intuitively obvious that this is a truth of logic. i.e. something which is valid on purely formal grounds”. In L. Åqvist 1984, p. 615 it is Axiom A2. In Kalinowski’s second-most classical paper on the logic of deontic propositions (J. Kalinowski 1953), molecular formulas of this form simply do not occur.

it is obligatory (that x do action q if she ⁶ has done action p) ⁷ then, if it is obligatory for her to p , then it is likewise obligatory for her to q .

2. ARE THERE INTERNALLY CONDITIONAL OBLIGATIONS?

Now it is unfortunately anything but self-evident that such conditional obligations as those of the form “for any agent x , it is obligatory (that x do action q if she has done action p)” – call them “internally conditional obligations” – do actually occur in any real-life deontic texts. Kalinowski, a legal scholar and a logician in one, thinks they do not ⁸. Sentences of this sort are not at all featured in Ferrajoli’s monumental third volume of his *Principia iuris* ⁹, devoted precisely to the formal-logical aspect of the legal discourse. In real-life deontic texts, such as legal codes, conditional obligations usually have the form “ $A \rightarrow OB$ ” ¹⁰, or “if something or other happens, certain agents will be under the obligation to do this or that”. Yet it would, perhaps, make nonetheless some sense to say that it is, for example, everybody’s (*prima facie*) duty to return borrowed money (if any), and that everybody is under this obligation always, not just only once one has, in fact, borrowed some money. For psychological as much as for educational

⁶ Inclusive pronouns used throughout this essay.

⁷ The (somewhat clumsy) parentheses are supposed to mark the scope of what is obligatory for x .

⁸ G. Kalinowski 1973, p. 56. Kalinowski criticises not just “ $O(A \rightarrow B)$ ” but, likewise, “ $P(A \rightarrow B)$ ”, “ $O(A \vee B)$ ” and “ $P(A \vee B)$ ” as unmet-with in real-life deontic (that is, moral treatises and legal codes, in the main) texts. Kalinowski’s wording is remarkable. Expressions like those count, he tells us, as well-formed in von Wright’s Old System (that is, von Wright’s classical paper, G.H. von Wright 1951) (and generally, we should add, in standard deontic logic) “*ohne daß man sich im vornhinein von der Existenz von Normen mit dieser syntaktischen Struktur überzeugt hätte. Befragt man nun post factum einen Ethiker oder Juristen dazu, so werden sie antworten: ‘Wir verstehen’ was es heißt: ‘Wenn p , dann OA ’ (als Beispiel), wobei ‘ p ’ für einen theoretischen [i.e., purely descriptive] Satz steht, der einen eine Verpflichtung bedingenden Sachverhalt beschreibt [...]. Wir verstehen die Bedeutung von: ‘Wenn OA , dann OB ’ [see note 10 in this essay]. Wir kennen auch die Ausdrücke, die durch Einsetzen in irgendeine Formel hervorgehen. Und wir verstehen den Sinn, den man ‘(Wenn A , dann B) ist obligatorisch’ geben möchte, aber wir müssen gestehen, bis jetzt die Normen mit dieser syntaktischen Struktur [nicht; this word is strangely lacking in the German translation] gesehen zu haben. Es ist allerdings wahr, daß wir nicht all die praktisch unzähligen moralischen und juristischen Normen durchmuster haben. Vielleicht haben wir sie bloß zufällig nicht erwischt”. Part of the problem is that we are not quite sure how to understand such formulas (see A.N. Prior 1954, p. 65), despite their apparent simplicity.*

⁹ L. Ferrajoli 2007.

¹⁰ For an example, see art. 6:21 and 6:22 of the Dutch Civil Code: “An obligation is conditional when, as a result of a juridical act, its effect depends on the fulfilment of an uncertain future event” (6:21); “A condition precedent makes that the obligation only takes effect when the uncertain future event sets in” (6:22).

reasons it would, perchance, be better to drive that point home to each and every potential money-borrower in this form, i.e. as a special case of the protasis of the Kripke-principle. In case you have not, or not yet, borrowed any money, well, you are entitled to take it easy, as you are discharging this conditional $O(A \rightarrow B)$ duty of yours by doing simply nothing.

3. CAN WE REALLY DISTRIBUTE THE “O” OPERATOR?

What happens, however, if we presuppose that given the above – i.e. “it is everybody’s (prima facie) duty, if they have borrowed money, to return it to the lender” –, Jane is additionally under the obligation to actually borrow some money, perhaps as the only means of providing for her family’s immediate needs or because of some other urgent and pressing circumstances? Is Jane, then, by the same token obliged to return “the” money¹¹? Certainly not, at least according to common sense; she will be so as soon as she discharges her obligation to borrow money, but definitely no sooner. It seems, perhaps, to make some sense to say that Jane even at the present moment ought (in the present tense) to return the money that she will have borrowed as a matter of discharging that former obligation, but the problem is that it is not at all certain, and, as experience teaches, it is a far cry from certain, that she will ever borrow any money. There are far too many factors that may keep Jane from doing her duty, not just her laziness and forgetfulness but also a great many others which cast no shadow on her profile as a duty-fulfiller (e.g. she may be taken ill).

Fiddling with time-indices and such-like will not be, either, to very much avail: a trick like that of saying that while Jane is *now* under the obligation to borrow money until, say, the day after tomorrow afternoon, she *is* no less *now* (and not any later) under the obligation to return the money she will have borrowed until the day after tomorrow afternoon does the job only and exclusively under the controversial and controvertible presupposition that Jane will, in fact, have borrowed the money until the day after tomorrow afternoon; which she may, for independent reasons, just as well not¹². If we are in no mood for quarrel, we may well concede that, okay, Jane is *even now* under the obligation to pay back the amount of money she will possibly, God kindly willing and under all sorts of duly

¹¹ An example possibly slightly more realistic and natural, and less far-fetched, than the ones (relevant for the same problem) provided by McLaughlin in R.N. McLaughlin 1955, p. 400.

¹² Ho wisely gives up the K-principle in his solution of the famous Chisholmian contrary-to-duty paradox (which itself arises due to the K-principle, see P. McNamara 2014, sec. 4.5), (N.D. Ho 1997, p. 98), although he is formal logician enough to retain it in his temporal-deontic logic system (N.D. Ho 1997, p. 97).

propitious circumstances, have borrowed ... but this amounts very much to $O(p \rightarrow q)$, where what we need is a plain robust Oq . And Jane's availing herself or not of the benevolence of a lender, should any come her way, is not a real *futurum contingens* in the sense of *De Interpretatione*, chap. 9, as she is truly bent, let's charitably suppose, on discharging her duties conscientiously and as far as humanly possible ... Yet still, Jane's having actually borrowed the money— so as there should actually be something to pay back — cannot be jumped to so as to reach her obligation to repay the loan, no matter how conscientious and self-disciplined Jane be, as there are far too many countervailing circumstances possible, of both factual and normative nature.

Again, it could, perhaps, be argued that Jane, being obliged to take out a loan, is therewith also obliged to pay back the loan, namely the loan she has possibly not yet asked for and perhaps never will, but which she is nonetheless obliged to take out. This seems to make some sense (is a loan not a loan, after all, regardless whether already granted or merely intended?), but then the question cannot fail to arise whether “ q ” is not ambiguous as between occurring in “ $O(p \rightarrow q)$ ”, on one hand, and in “ $O(q)$ ” on the other; in the former, it symbolises “repaying money actually borrowed” whereas in the latter it symbolises “repaying the money one is under the obligation to borrow”.

4. WHY THE KRIPKE SEMANTICS LEADS US ASTRAY

I think part of the explanation why we often fail to make such seemingly simple observations¹³ as the above is that we are all under too heavy an influence of the Kripke semantics for deontic logic. The K-principle is obviously true in any Kripke-semantics model of any standard deontic logic: If in every accessible (deontically perfect) world $p \rightarrow q$ holds and in addition in every such world p holds as well, then obviously in every such world q holds, too. But the Kripke semantics is, as I should like to submit, ill-suited for the purpose of making semantic sense of deontic propositions as normally understood. It is a formidable instrument, true, for modal logic and perhaps a great many other non-classical logics, but for deontic logic it is all but useless.

¹³ Well, logicians fail to make them because they are primarily interested in their formal systems, not in whatever someone might presumptuously think these are supposed to be applicable to. To paraphrase Wilfrid Hodges' Hilbert: “Wir sind Logiker” (W. Hodges 1983, p. 75). The original Hilbert, “*Erinnern wir uns, daß wir Mathematiker sind*” (D. Hilbert 1926, p. 174) is less jaunty.

In particular, that all agents or just Jane are under the obligation to do p does not mean (unless this be violently and arbitrarily pressed into the Procrustean bed of the Kripke semantics) that in all worlds deontically “accessible”, “acceptable” or “ideal”, whatever you prefer to call it, from the point of view of this world the agents in question actually do p , or actually discharge their duty. They sometimes do and sometimes do not, for in some of such worlds, there might be all kinds of *forces majeures* and other mitigating circumstances, as well as overriding norms¹⁴, conspiring to make these worlds both deontically accessible (or, if you will, “acceptable” and “ideal”) and yet not “perfect” in the sense of everyone’s conscientiously discharging one’s duty. “England expects that every man will do his duty” in order to impose on every man another duty, but sometimes she is expecting in vain, as some men die before they have done theirs. Jane may be thwarted at achieving her duty-conforming purpose of borrowing money by no weaker reason than the absence of a suitable and willing lender, which, again, may not be due to anybody’s unexcused breaching *their* duty, the relevant world being as deontically “acceptable” as can possibly be¹⁵.

5. A MODEST PROPOSAL OF AN ENTIRELY THIS-WORLDLY SEMANTICS AND ONTOLOGY FOR DEONTIC PROPOSITIONS

Semantics of propositions is about truth and falsity; now norms are true or false¹⁶ not in virtue of matters of fact in “accessible (deontically accepta-

¹⁴ See, e.g., G. Kalinowski 1973, p. 57.

¹⁵ This consideration shows that the Kripke semantics is, by itself, not the real “bad guy” to vilify here; the fault seems to be, rather, with the rather ingenuous – or should we rather say, spelling deontic innocence – use that is often made of it, plus an uncritical use of the “Ought implies Can” (cf. G. Feis 2015) principle. A deontically accessible world (in S. Kripke’s sense) need not at all be one in which all obligations are fulfilled, as there may be impediments which are absolutely no-one’s fault and beyond anyone’s control. Yet on the other hand, against a literal reading of the “Ought implies Can” principle, such impediments typically do not cancel the obligation. They only excuse its breach. Clearly, a world without even excused or excusable breaches of obligations is a deontically “more ideal” world than any world where, like in ours, such breaches are abundant; on the other hand, it remains questionable if such a world is a “possible” one in any interesting sense – that is, such as to render a logic whose semantics is based in the notion of a world like that any useful for controlling and logically disciplining “real-life” deontic discourse. A preference semantics (starting from about B. Hansson 1969, thanks go to Professor Urbaniak of Gdańsk University for drawing my attention thereto) may be a suitable instrument for fine-tuning all Kripke-inspired semantics in the respects here relevant.

¹⁶ Yes, I do know that contemporary philosophy’s unshakeable dogma is that norms are neither true or false; hence the famous or infamous Jørgensen dilemma (P. McNamara 2014, sec. 4.1). I have tried to voice some opposition in W. Żelaniec 2015 and W. Żelaniec 2008. The dogma, though it certainly contains a grain of truth, is false as it stands, I have argued. For an excellent study of the various surrogates of the good old

ble) worlds” being thus-and-so, I should like to propose, but in virtue of a *sui generis*¹⁷ relationships between classes of agents and classes of states of affairs, both of them of and in *this* world. The obtaining or non-obtaining of these relationships themselves could then be called “normative states of affairs”. The obligation to return borrowed money is, for instance, a *sui generis* relationship between the class of borrowers and the class of states of affairs (of the same world!) in which the borrower returns the money to the lender. Since classes are abstract beings and we are dealing with both actual and potential borrowers and actual as well as merely possible (conceivable) money-returnings to both actual and merely potential money-lenders, the temptation may arise to start thinking about such matters in terms of possible worlds and such-like. But we had better resist all such temptation.

For, although very *sui generis*, the deontic relationships between agents and states of affairs are no “weirder” ontologically, and no more in need of an interpretation in terms of possible worlds, than are, say, those of “standing a good chance of becoming of achieving something or other”, or those of “being jeopardised by”. Many persons and other objects stand a good chance of achieving this or that, this seems to me an entirely this-worldly truth, many things are jeopardised by various (actual or merely potential) events or processes¹⁸, not of virtue of any, yes, hard facts, yet in other (possible) worlds – but in virtue of this world, its facts but also its propensities

truth-value for norms see G. Lorini 2003. This anthology reflects quite well Professor Lorini’s years of study of the problem of norms: G. Lorini - L. Passerini Glazel 2012. See, too, G. Kalinowski 1967 and A. Pintore 2000 as well as A. Marmor 2011. And, lest you should think that I am ignoring the most obvious: norms as norming speech-acts (norms in the act of being promulgated) are *not* or not *per se* true or false. The proper reaction to a legislator’s proclamation of the form “you may/ought to/must not *p*” is acceptance (acquiescence) or rebellion, or ignoring perhaps, but certainly not asking whether the speech-act be true or false. (The famous Anscombian-Searle’an “direction of fit” – G.E.M. Anscombe 1958, pp. 56 f., sec. 32 and I.L. Humberstone 1992 – is from speech-act to states of affairs, not the other way ’round.) Afterwards, however, it is very much possible and meaningful – is it not? – to ask whether (it is true that) one may/ought to/must not *p*. However, in the present context these knotty problems are not really topical; for our purposes it will be admissible to treat truth-values of norms as though they were identical with the truth-values of the corresponding *Rechtssätze* in late Kelsen’s sense (see E. Bulygin 2013, p. 226; G.N. Dias 2005, p. 133 note 7 and R. Walter 1993, p. 350), i.e. the purely descriptive propositions stating the validity of the corresponding norms. In this way, the question whether it is true that you may/ought to/must not *p* collapses to the question whether the corresponding norm is in force. For a similar use of Hedenius’ “spurious deontic propositions” in contemporary standard deontic logic see L. Åqvist 1984, p. 614, cf. Hansson 1969, p. 375.

¹⁷ And it is one of the hardest questions of philosophy to establish what that *genus* is.

¹⁸ For instance, we are told that astronauts’ – a very interesting topic to those of us, given the recent rise of space tourism, who find bungee-jumping or scuba-diving too pedestrian – performance is easily jeopardised by sleep-deprivation, K. Collins 2014. Probably not just astronauts’ ...

and the scientific laws that govern it. Or take “it is easy for x to do p ”, which is clearly another member of the same family: certain *sui generis* relationships between classes of potential agents and classes of actions (of the same world) such that it is easy for these agents to perform these actions. By way of an example, and in a somewhat idealist mood, we could say that it is easy to get a prestigious and well-paid job after having graduated *summa cum laude* from an excellent university (that is, for any x , it is easy if she has graduated *summa cum laude* from, say, “La Sapienza”, or the “Julius-Maximilians-Universität” Würzburg), for her to get a good job position afterwards), and then for some persons it may be in fact easy to graduate *summa cum laude* from a top-notch institution of research and higher education – but will it, even *before graduation*, for the self-same persons be easy to get started on a brilliant professional career? It may, but need not, be; logic alone cannot presume to warrant it, for the simple reason that doing two easy things in direct succession may not be an easy feat.

The Kripke-principle wrecks proof-theoretic havoc with the standard deontic logic itself, in that it generates most if not all of its famous or infamous paradoxes¹⁹. But this need not detain us here, for, remember, we are no deontic logicians. We have only been concerned with model-theoretic considerations and the upshot of these is that the K-principle has to be employed, if at all, very cautiously in any attempt at making deontic logic any useful for controlling real-life deontic discourse.

6. LASTLY: HOW CAN THE K-PRINCIPLE BE WRONG IF IN VON WRIGHT IT IS A TAUTOLOGY?

In von Wright (1951) the K-principle is indeed a tautology. As such it cannot, obviously, be wrong. Let’s see why von Wright thinks it is a tautology.

To cut a long story short, von Wright unobjectionably represents (p. 12) the K-principle in this form: $\sim P(\sim A) \ \& \ \sim P(A \& \sim B) \ \rightarrow \ \sim P(\sim B)$. Next, he sets about reconstructing the disjunctive normal form of the names of the actions inside the P-operator, which he proposes are as follows:

1. $\sim A$: $\sim A \& B \vee \sim A \& \sim B$
2. $A \& \sim B$: $A \& \sim B$
3. $\sim B$: $A \& \sim B \vee \sim A \& \sim B$

Next, he appeals to his Principle of Deontic Distribution (p. 7), which states that $P(\alpha \vee \beta) \leftrightarrow P(\alpha) \vee P(\beta)$. With it, and given the above form of the

¹⁹ See P. McNamara 2014, from section 4.3 onwards, starting from the Alf-Rossian “Free Choice Permission Paradox” (in its derivation the principle called by McNamara “OB-RM” is used: $\vdash p \rightarrow q$ then $\vdash Op \rightarrow Oq$, for the derivation of which, in its turn, the K-principle is indispensable). Cf. note 12.

K-principle as well as 1.-3., the K-principle can be reduced, by means of trivial transformations, to this: $\sim P(\sim A \& B) \& \sim P(\sim A \& \sim B) \& \sim P(A \& \sim B) \rightarrow \sim P(A \& B) \& \sim P(\sim A \& \sim B)$, which is, clearly, a tautology ($p \& q \& r \rightarrow r \& q$). (Let's call it "NF" for short, the first letters of "normal form".)

This is in perfect logical order except that ... conjuncts starting with $\sim A$ make no sense if A stands for borrowing money and B for returning the borrowed money. If Jane has borrowed no money, she cannot either return or not return "the" money, as there is no "the" money. The two actions are not logically and ontologically independent. As McLaughlin has proposed (1955, p. 400), the truth-value of the K-principle depends on the "performance value" of A , and my example illustrates this very well.

McLaughlin's own "smoker's" example (p. 400) does not present exactly the same kind of difficulties. Suppose, he says, that a norm commits us to smoking in the smoking compartment, if we are smoking (such norms were usual in the old days when smoking was at all permitted), and suppose, he goes on, that a separate norm makes smoking obligatory during a train trip. Now a defiant train traveller refuses to smoke – is he nonetheless obliged to enter the smoking compartment? If A stands for smoking, and B for being in a smoking compartment, then obviously, as distinct from the money-borrower case, there is an action such as $\sim A \& B$ or $\sim A \& \sim B$: you *can* very well not be smoking and yet be or not be in the smoking compartment. So McLaughlin's is not at all a counterexample to the K-principle?

However, this is an illusion which rests on an ambivalence and an ambiguity, reflecting too crude an understanding of the norms in hand. The former norm (the one which says that smoking commits one to being in a smoking compartment and which we should like to symbolise as $O(A \rightarrow B)$) does not mean, literally, that on catching oneself smoking one is obliged to see to it that one should in no time find oneself in the smoking compartment. Such things happen in Science Fiction, perhaps, but not in real life. Neither does it mean that an inadvertent smoker should at once find and enter a smoking compartment. It does mean, it could be said, that smoking is admissible only in smoking compartments, but given the structure of human agency this amounts to the following: anticipating smoking (for whatever reasons, from yielding to temptation to obedience towards another norm) commits to finding, *prior to smoking*, a seat in a non-smoking compartment. By contrast, the latter norm ($O(A)$) does not say: "you ought to anticipate smoking while aboard of the train", but a plain "you ought to smoke on the train". We have thus $O(C)$, rather than $O(A)$ here, alongside $O(A \rightarrow B)$, and the Kripke principle simply does not find application²⁰.

²⁰ A really interesting question would arise if the second norm went, not "you ought to smoke ..." but "you ought to anticipate smoking ...". It is clearly possible not to anticipate smoking and yet (not) be in the smoking compartment, so the formula NF

The reasonings above do not, obviously, invalidate the K-principle logically; it is and remains a tautology – at least within the framework of von Wright 1951. What they do show, I claim, is that the proper application of this principle to specific cases is not a matter of course.

REFERENCES

- Anscombe, G.E.M. (1958). *Intention*, Oxford: Blackwell.
- Åqvist, L. (1984). Deontic Logic, in D.M. Gabbay - F. Guenther (eds.), *Handbook of Philosophical Logic*, vol. 2, Dordrecht: Kluwer, pp. 605-714.
- Bulygin, E. (2013). Kelsen on the Completeness and Consistency of Law, in L. Duarte - J. Gardner - L. Green (eds.), *Kelsen Revisited: New Essays on the Pure Theory of Law*, Oxford, Bloomsbury Publishing, pp. 225-243.
- Collins, K. (last). *Astronaut Performance Jeopardised by Sleep Deprivation*. <http://www.wired.co.uk/news/archive/2014-08/08/astronauts-sleep-deprivation>. Retrieved 8. Aug. 2014.
- Dias, G.N. (2005). *Rechtspositivismus und Rechtstheorie: das Verhältnis beider im Werke Hans Kelsens*, Tübingen: Mohr Siebeck.
- Feis, G. (2015). *Impossibilità nel diritto*, Pisa: ETS.
- Ferrajoli, L. (2007). *Principia iuris. Teoria del diritto e della democrazia*. 3. *La sintassi del diritto*, Roma - Bari: Laterza.
- Hansson, B. (1969). An Analysis of Some Deontic Logics, *Nous*, vol. 3, n. 4, pp. 373-398.
- Hilbert, D. (1926). Über das Unendliche, *Mathematische Annalen*, vol. 95, pp. 161-190.
- Ho, N.D. (1997). On the Dilemma of Conditional Obligation, in G. Meggle - A. Mundt (eds.), *Analyomen 2. Proceedings of the 2nd Conference Perspectives in Analytical Philosophy*, vol. 1, Berlin - New York: Walter de Gruyter, pp. 93-100.

$(\sim P(\sim A\&B) \& \sim P(\sim A\&\sim B) \& \sim P(A\&\sim B) \rightarrow \sim P(A\&B) \& \sim P(\sim A\&\sim B))$ does seem to make sense in this case. This is a very far-fetched example and I cannot discuss it here at length. In a nutshell, my answer would be that it all depends on what “anticipate” should here be taken to mean. If it means “intend”, then “you ought to anticipate smoking ...” is an impossible norm (you cannot legislate on anybody’s intentions, except, perhaps, indirectly). If it means “acknowledge that as a matter of inevitable fact you will (smoke)”, then it is, too, an impossible norm as you cannot, still less than on intentions, legislate on insights (not any more on your own than on other people’s); yet, the legislator, seeing that the passengers (inveterate and passionate smokers, let’s assume) will sooner or later yield to temptation no matter what they now fail to acknowledge, will, issuing his norms, countenance the impossibility of legislating on insights by grouping the last two conjuncts of the protasis of NF in such a way as to make them mean: “you ought to seat yourself in the smoking compartment in any case, whatever you acknowledge or fail to acknowledge” (just as if you were saying: “you really ought to rinse the dishes, whether or not you can square the circle”). Which is equivalent to $O(B)$.

- Hodges, W. (1983). Elementary Predicate Logic, in D.M. Gabbay - D.M. Guentner (eds.), *Handbook of Philosophical Logic*, vol. 1, Dordrecht: Reidel, pp. 1-131.
- Humberstone, I.L. (1992). Direction of fit, *Mind*, vol. 10, n. 401, pp. 59-83.
- Kalinowski, G. (1967). *Le problème de la vérité en morale et en droit*, Lyon: Vitte.
- Kalinowski, G. (1973). *Einführung in die Normenlogik*. English translation by W. Klein, Frankfurt am Main: Athenäum.
- Kalinowski, J. (1953). Teoria zdań normatywnych / Theorie des propositions normatives, *Studia Logica: An International Journal for Symbolic Logic*, vol. 1, 113-184.
- Lorini, G. (2003). *Il valore logico delle norme*, Bari: Adriatica.
- Lorini, G. - Passerini Glazel, L. (eds.) (2012). *Filosofie della norma*, Torino: Giapichelli.
- Marmor, A. (2011). *Truth in Law*. <http://law.bepress.com/usclwps-lss/art73/>. Retrieved 20. Nov. 2014.
- McLaughlin, R.N. (1955). Further Problems of Derived Obligation, *Mind*, n.s., vol. 64, n. 255, pp. 400-402.
- McNamara, P. (2014). Deontic Logic, in E.N. Zalta (ed.), *The Stanford Encyclopedia of Philosophy*. <http://plato.stanford.edu/archives/spr2014/entries/logic-deontic>.
- Perelman, C. (1968). Beitrag zu: *Akten des XIV. Internationalen Kongresses für Philosophie (Wien, 2.-9. September 1968), Band II*, Wien: Herder, pp. 269-311.
- Perelman, C. (1970). Interventions au Colloque de Bruxelles, in C. Perelman (ed.), *Études de logique juridique. Le raisonnement juridique et la logique déontique. Actes du Colloque de Bruxelles (22-23 décembre 1969)*, Bruxelles: Émile Bruylant.
- Pintore, A. (2000). *Law without Truth*, Liverpool: Deborah Charles.
- Prior, A.N. (1954). The Paradoxes of Derived Obligation, *Mind*, n.s., vol. 63, n. 249, pp. 65-65.
- von Wright, G.H. (1951). Deontic logic, *Mind*, n.s., vol. 60, n. 237, pp. 1-15.
- Walter, R. (1993). Das Problem des "normativen Syllogismus" in Kelsens "Allgemeiner Theorie der Normen", in W. Krawietz - J. Wróblewski (eds.), *Sprache, Performanz und Ontologie des Rechts. Festgabe für Kazimierz Opalek zum 75. Geburtstag*, Berlin: Duncker und Humblot, pp. 347-355.
- Żelaniec, W. (2015). Is a Truth-value of Norms Really as Nefarious an Idea as All That?, in P.L. Lecis - V. Busacchi - P. Salis (eds.), *Realtà, verità, rappresentazione*, Milano: FrancoAngeli, pp. 351-365.
- Żelaniec, W. (2008). *The Truth-value of Norms*. http://blog.centrodietica.it/wp-content/uploads/2010/02/truth_value_o_norms.pdf. Retrieved 20. Nov. 2013.