Journal of Educational, Cultural and Psychological Studies

18
December 2018

Studi e Contributi di Ricerca
—
Studies and Research Contributions

Note di Ricerca
Research Notes

Didactic Contract as a Key to Interpreting Gender Differences in Maths

Giorgio Bolondi - Federica Ferretti - Chiara Giberti *

Libera Università di Bolzano - Faculty of Education (Italy)

DOI: http://dx.doi.org/10.7358/ecps-2018-018-bolo

chiara.gib@gmail.com
federica.ferretti5@gmail.com
giorgio.bolondi@unibz.it

IL CONTRATTO DIDATTICO COME UNA CHIAVE DI LETTURA PER INTERPRETARE LE DIFFERENZE DI GENERE IN MATEMATICA

Abstract

Research, at both national and international levels, has started to focus on the gender gap regarding student performances in technological/scientific topics. Fervent ideological and epistemological debates have sprung up, particularly concerning the differences in mathematics test results between males and females (with males outshining females in most countries). In this paper, we analyse some questions taken from the INVALSI mathematics tests administered at varying educational stages; emergent didactic phenomena are investigated via one of the key concepts of mathematics education, the didactic contract. From a quantitative analysis, based on the percentage of answers given and Rasch Model results, it is possible to identify the presence of significant gender differences between the performances which can be traced back to the effects of the didactic contract. Furthermore, study of the features which accompany these gender gaps in the case studies allows us to investigate the gender gap in mathematics via a new interpretative method which also takes into account student ability levels.

---

* The contribution is the result of the joint work of the three authors. In particular, Giorgio Bolondi edited the paragraph 1. Federica Ferretti edited the paragraphs 3, 5.1 and 6. Chiara Giberti edited the paragraphs 2, 4 and 5.2.

Keywords: Didactic contract; Gender gap; Mathematics education; Rasch analysis; Standardized assessment.

1. Introduction

In recent years, an increasing number of research studies have focused on differences in math performances between males and females. The growing importance of international standardized assessment systems such as OECD-PISA and IEA-TIMMS, and the introduction of national surveys, have provided a new emphasis on this issue that calls into question not only education but also other viewpoints such as politics, economy, and psychology.

International surveys reveal that Italy is one of the countries with a more significant gender gap in maths (Mullis *et al.*, 2016; OECD, 2016b;), and this is also confirmed by Italian standardized assessment INVALSI tests, administered every year at different educational stages (INVALSI, 2016).

In line with data published in the Ministerial guidelines, analysis of INVALSI test results and their interpretation as given by SNV experts provide an overall picture not only of the educational system but also of specific weaknesses of students (detectable by standardized assessment). In this perspective, standardized assessment data are an excellent resource for anyone who deals with «school». Analysis of the test results whilst taking into account context information (such as socio-economic background, gender, or geographical area) is proving to be an essential tool for the improvement of educational practices (e.g. Alivernini *et al.*, 2017).

Many research projects centred on international and national standardized assessment results have tried to explain the determinant of gender differences in mathematics, but many of them have focused only on overall results of the whole test.

In this paper we study gender differences focusing on specific items and this item-level analysis has led us to interpret gender differences through the lens of mathematics education. According to Leder and Lubiensky (2015), «it is important for item-level analyses to be systematically conducted and reported» in order to «pinpoint the mathematics that students do and do not know», gain a clearer understanding of disparities between groups, and «inform both textbook writers and teachers, as they strive to address curricular areas in need of additional attention».

In particular, our research aims to highlight the possible presence of gender differences in specific questions in relation to the concept of didactic contract, and therefore understand whether didactic contract has a different influence on males and females.

2. Gender gap in mathematics

Gender gap in maths is a popular area of study in educational research, and interest in the question has increased significantly over the last few years due to the growing importance awarded to standardized assessment on both national and international scales (Leder & Forgasz, 2008).

Results of large-scale international assessment tests, such as OECD-PISA and IEA-TIMSS, highlight differences between male and female performance in mathematics: males perform better than females at almost all school levels (Mullis *et al.*, 2016; OECD, 2016a) and this is confirmed by all surveys of recent years.

Moreover, it is interesting to note how international surveys reveal that this gap is not equally distributed worldwide: in the majority of countries, the gender gap is in favour of males and statistically significant, but there are also countries in which the gender gap is not significant, and yet others where females outperform males (Mullis *et al.*, 2016; OECD, 2016a).

In this context, Italy results as one of the countries in which the gap is larger, even though the overall performance of Italian students approaches the total mean score of the countries concerned (OECD, 2016b). This means that in Italy girls perform far below the PISA and TIMSS average, while boys perform far above said average (Mullis *et al.*, 2016; OECD, 2016b). The existence of a significant gender gap in maths in Italy is also confirmed by national standardized assessment tests, known as INVALSI, administered every year to all students attending grades 2, 5, 8 and 10. Indeed, the results of INVALSI tests revealed that in Italy the gender gap exists at all scholastic levels and, according to other studies based also on international results, it increases throughout the years at school (INVALSI, 2016; Contini, Di Tommaso, & Mendolia, 2017).

The causes of a gender gap in mathematics are diverse and their interrelation is complex; many studies have tried to explain this phenomenon from different perspectives. Firstly, according to other research studies (Leder, 1992; Gallagher & Kaufmann, 2005; Winkelmann, van den Heuvel-Panhuizen, & Robitzsch, 2008) we can identify internal factors which are directly related to the individual, such as biological causes, cognitive abilities and psycho-social factors.

Among these factors, the role of biological causes seems to be of limited importance, particularly with reference to the results of international large-scale assessment: indeed, as stated previously, gender gap in maths is not equally distributed in all the countries and there are also countries with no gap. Thus, biological causes cannot be ascertained as one of the determining factors (Hill, Corbett, & St Rose, 2010; OECD, 2016a; Contini *et al.*, 2017).

Furthermore, differences in cognitive abilities between boys and girls do not appear to be very relevant in explaining this phenomenon. Many studies argue that differences in general cognitive abilities are not significant (Halpern, Beninger, & Straight, 2011; Ruffing *et al.*, 2015) and even if a disparity exists, it can be compensated with brief and focused training (Hill *et al.*, 2010).

On the other hand, differences in metacognitive aspects related to maths provide an interesting viewpoint on this topic. From primary school onwards, girls display less confidence in their abilities, less math self-efficacy, less math self-concept and higher levels of math anxiety, even if achieving comparable results with boys in school (Fredericks & Eccles, 2002; Herbert & Stipek, 2005; Pajares, 2005; Cargnelutti, Tomasetto, & Passolunghi, 2016; OECD, 2016a). From our perspective, it is interesting to note that in OECD PISA results concerning self-efficacy (OECD, 2015), the gap between boys and girls is smaller in the tasks that explicitly ask pupils to solve equations. Such tasks can be solved using a routine procedure studied earlier in classroom practice, and this may encourage girls.

Moreover, girls seem to be better in discipline, compliance with rules, participation in schoolwork and self-control (Matthews, Ponitz, & Morrison, 2009; OECD, 2015). This can favour girls' school performance but may sometimes lead them to be excessively tied to classroom practices.

Alongside the internal factors, we also need to consider various external factors contributing to the gender gap in maths.

The fact that the gender gap is not equally distributed in all countries suggests that social and cultural factors must be one of the main determinants of gender gap in mathematics. Indeed, different studies have shown that in countries with a more gender-equal culture, gender differences in maths test scores decrease or even disappear (Jacobs & Eccles, 1992; Guiso *et al.*, 2008; González de San Román & De La Rica, 2012; OECD, 2015; Cascella, 2017).

Moreover, gender stereotypes imposed by society, and the beliefs of teachers/parents concerning boys' and girls' maths skills have a significant impact on students' self-perception and consequently on their performances (Jacobs & Bleeker, 2004; Riegle-Crumb, 2005; Freyer & Levitt, 2010).

Studies have also considered other factors related to the school context, such as curriculum variables (Leder, 1992), classroom practices, assessment methods and teaching methods (Leder & Forgasz, 2008; Giberti, Zivelonghi, & Bolondi, 2016; OECD, 2016a; Bolondi, Cascella, & Giberti, 2018). In this perspective, researchers have also focused on the different strategies used in problem solving activities, pointing out that females apply routine procedures and well-known strategies more frequently than males. The explanations for this different behaviour are inherent to metacognitive aspects: males seem less worried about being wrong and more inclined to try new methods and alternative approaches (Gould, 1996; Fennema & Carpenter, 1998; Gallagher *et al.*, 2000; Bell & Norwood, 2007).

Therefore, the link between gender gap in mathematics and socio-cultural factors is strong, and to understand this phenomenon we also need to consider micro-social factors related to the classroom environment: classroom practices, curriculum variables, relationships with the teacher and with the discipline itself. Thus, in this study we set out to investigate whether the didactic contract may be a suitable theoretical lens to analyse specific features of gender gap.

3. Didactic contract and gender gap

The concept of didactic contract emerged in studies by G. Brousseau, carried out in France at the end of 1970 and aimed at investigating the causes of failure (particularly of elective failures) of mathematics students (Bordeaux IREM, 1978). The construct lends itself very well to interpreting many aspects of classroom situations wherein mathematics is performed, and it has become one of the most well-known and widespread constructs for international literature in mathematics education.

The three main players in the didactic contract are the teacher, the student and the knowledge, and one of its most widespread definitions is as follows:

> the set of the teacher's behaviours as expected by the student, and the set of student's behaviours as expected by the teacher. (Brousseau, 1980a, p. 127)

In Brousseau's studies, mathematical problems are investigated, and the causes are directly connected to the learning/teaching process (Brousseau, 1980a, 1980b). The didactic contract is an interpretation of the presumed commitments, mutual expectations, and sanctions foreseen by one of the protagonists of a didactic situation for him/herself and for each of the others, closely linked to mathematical knowledge (Brousseau, 1997). As we can see in the previous paragraph, studies show that females seem to be better in discipline, compliance with rules and participation in schoolwork (Matthews *et al.*, 2009; OECD, 2015); nevertheless, girls have lower confidence in their abilities and less maths self-efficacy. This is due to their being particularly tied to classroom practices and at the same time «insecure» about their abilities and efficacy; it seems to imply the existence of a «stronger» didactic contract among females. The question thus arises of whether (and how) this is related to the gender gap. A lot of research studies, (e.g. OCSE, 2015; Giberti *et al.*, 2016), have shown that the main reasons for the observed differences in the performance of males and females in mathematics are related to the cultural and social context in which the students live. We also consider that micro-social factors related to milieu, curriculum variables and classroom practice might be key to understanding the issue of gender gap. In fact, these characteristics of the females lead to them relying more on the relationship with the teacher and the discipline, and then «delegating» their learning process to these two didactic players.

In general, is there a connection between gender gap and didactic contract? Specifically, when there are effects of didactic contract, is there a gender gap? As students' progress through their school life, the didactic contract takes on different aspects and seems to endure and, sometimes, to become even «stronger» (De Vleeschouwer & Gueudet, 2011; Ferretti, 2015). In the situations that we will analyse, will there also be changes with regard to the gender gap?

4. Methods

To carry out the research study, we studied some INVALSI tests with different quantitative and qualitative approaches; we investigated some tasks linked to the didactic contract, and we studied gender differences in these items.

We based our analysis on INVALSI tests both to obtain statistically valid and representative data of the population, and because the Rasch Analyses (Rasch, 1960) have allowed us, as we shall see later in detail, to investigate the research issues in depth.

INVALSI tests are administered every year in Italy to all students at grades 2, 5, 8 and 10. For each test, and therefore each grade, we analysed data of the INVALSI statistical sample which consists in approximately 30,000 students, and for this sample the tests were administered under controlled conditions.

The INVALSI math tests covers all content areas (Numbers, Space and Shapes, Statistics and Probability, Relations and Functions) and are composed of multiple choice or open-ended tasks.

Previous research studies on INVALSI tests evidenced that gender gap is not equally distributed across all the items: there are some specific items in which the gender gap is wider (Bolondi, Cascella, & Giberti, 2017). To analyse the significance of the gap in the correct answers to each item, we use a specific index (IGG presented in Bolondi *et al.*, 2017) which is calculated by considering the difference between the correct answers of the two groups as compared with the difficulty of the item.

$$IGG_k = \frac{M_k - F_k}{P_k}$$

In which:

- M_k is the percentage of correct answers from males to item *k*
- F_k is the percentage of correct answers from female to item *k*
- P_k is the percentage of correct answers from the whole population to item *k*.

In our research study, we analyse data using the Rasch Model, in line with INVALSI data analysis (INVALSI, 2017). Rasch Model belongs to Item Response Theory and is a simple logistic model (Rasch, 1960; Barbaranelli & Natali, 2005). It allows us to analyse an entire test by estimating an ability parameter for each student and a difficulty parameter for each item (delta).

In particular, we will focus on specific graph outputs of the Rasch Model, known as distractor plots (*Fig. 1*).

The distractor plot of a specific item reports the Characteristic Curve of the item (blue continuous line) which is the curve outcome of the model and explains the probability of choosing the correct answer as a function of the students' ability measured across the entire test.

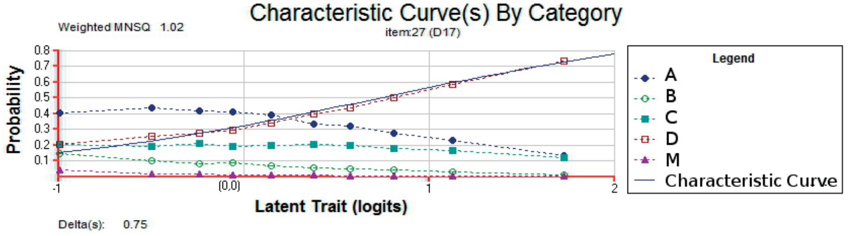

Figure 1. – Example of distractor plot. On the x-axes we have the ability of the students estimated by the Rasch Model on the bases of the entire test, on the y-axes we have the probability/percentage of choosing each answer.

In addition, distractor plots report the empirical data (dotted lines) relative to all the possible responses. In particular, they show the trend of the correct answer (growing dotted line) as a function of student ability across the whole test (students' ability parameter) whilst also revealing the trend of the incorrect answers.

In this way, it is possible to observe how students choose the correct answer and the probability that a student with a certain ability level (measured across the entire test) will respond correctly to the item. Furthermore, it is interesting to observe how students make mistakes and choose the other options. Distractor plots reveal that there are some incorrect answers (often linked to a specific resolution process) that prove more attractive to students belonging to a particular ability level, and this is particularly interesting to investigate through the theoretical lens of mathematics education (Ferretti, Giberti, & Lemmo, 2018).

In this paper, we analyse distractor plots of specific items strictly related to didactical contract issues. To study differences between male and female performances in these items, we plot distinct distractor plots for the two subgroups.

In order to have complete comparability of the two plots, we apply the Rasch Model to the whole population and estimate the ability parameter for each student. On the basis of these parameters (which place male and female ability on the same scale) we then construct the distractor plot of the two subgroups and compare them.

5. The data analysis

In this paper we present an item-level analysis focussed on two INVALSI tasks from different grades in which students' behaviour is attributable to the effects of the didactic contract, and we investigate gender differences in answering these items, again making reference to effects of the didactic contract.

5.1. *Analysis of Grade 10 INVALSI test*

The following question (*Fig. 2*) was administered in the INVALSI maths test in 2013 for Grade 10 students. The task was administered to 560,487 second-year students in the second cycle of each scholastic path and the sample consisted of 38,533 students, chosen according to criteria of representativeness (INVALSI & SNV, 2013).

D6. A hydrogen atom contains a proton whose mass m_p is approximately 2×10^{-27} kg and an electron whose mass m_e is approximately 9×10^{-31} kg. Which, of the following values, better approximates the total mass of the hydrogen atom (namely $m_p + m_e$)?

A. ☐ $2 \cdot 10^{-27}$ kg

B. ☐ $11 \cdot 10^{-31}$ kg

C. ☐ $11 \cdot 10^{-58}$ kg

D. ☐ $18 \cdot 10^{-58}$ kg

Figure 2. – Task D6 - Mathematics INVALSI test of 2013, Grade 10 students.

This question is included in the «Number» content area and requires students to estimate the result of the sum of two numbers expressed in scientific notation - they therefore have to operate with powers of ten. The correct answer is A, and the percentage of each options are reported in the table below (*Tab. 1*).

Table 1. – The results of task D6 in the Mathematics INVALSI test of 2013, Grade 10 students.

	Number of students	Percentage of students
A (correct)	**6893**	**18%**
B	3607	9%
C	12761	33%
D	12764	33%
M (missing)	2508	6%

As the results show, only 18% of the answers were correct. Analysing the percentages of the other options chosen, and the distractor plot of this task (*Fig. 3*), we can see that options C and D (in which the exponent of power is obtained by summing the exponents of the powers present in the text) were the most commonly chosen at national level (33% each) and were preferred by students at almost all levels of competence (*Fig. 3*); in fact, only for the highest decile did the percentage of students choosing the correct answer exceed the percentage of these two options.

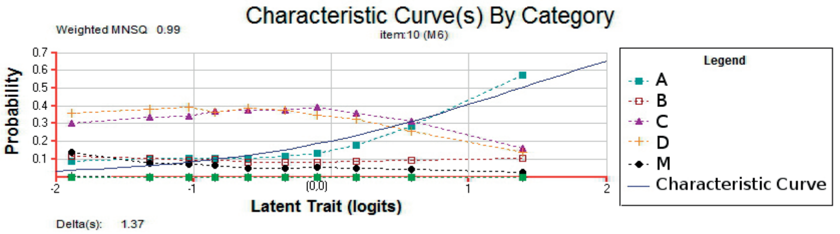

Figure. 3. – Distractor plot related to task D6, Mathematics INVALSI test of 2013, Grade 10 students.

The first interpretations (e.g. Impedovo, Orlandoni, & Paola, 2011) of the phenomenon highlighted by the task have linked the behaviour of pupils in general to effects of the didactic contract as outlined by Brousseau (1988) and to the lack of management of contents.

In particular, the fact that most of the students in the task, which intuitively recalls an additive situation, chose the distractors C and D (in which the sum of the exponents of the powers appear), seems to clearly indicate the presence of the clause of the didactic contract known as formal proxy (D'Amore, 2008). Moreover, the fact that the correct answer is a data explicitly present in the text has been studied in Ferretti (2015); this effect is called «The Age of the Earth Effect» and this research study highlights the fact that tasks with this feature command a very low percentage of correct answers.

Regarding male and female performances, as reported in the following table, we can see that this is a task with a remarkable gender gap in favour of males (*Tab. 2*).

Table 2. – Results divided by gender for task D6, Mathematics INVALSI test of 2013, Grade 10 students.

	All Students	Females	Males
A (correct)	**18%**	**14%**	**22%**
B	9%	9%	10%
C	33%	35%	32%
D	33%	36%	31%
M (missing)	6%	7%	5%
Gender Gap Index = 49%			

Observing the percentages of correct answers, we have an 8-percent gap in a really difficult question and this is evidenced also by an extremely marked Gender Gap Index.

It is particularly interesting to notice that this gap is principally due to the two options (C and D) analysed earlier. This may indicate that females are more affected by the didactic contract effect and choose option C and D by trying to apply some kind of power properties even though the task resolution requires a sum of powers (Giberti *et al.*, 2016). This phenomenon is widely studied in the literature and is an element of the didactic contract known as *formal proxy* (D'Amore, 2008). This fact can be related to the fact that, as explained before, girls display less self-efficacy and are more confident only in solving routine problems and applying well-known-algorithms.

Moreover, we can also observe a difference in missing answers (7% of females and 5% of males); this may confirm that males are more confident and display higher levels of self-efficacy, particularly in non-routine tasks.

Before progressing, we can also analyse gender differences with regard to students' ability across the whole test, comparing distractor plots for males and females as in the following figures (*Figs. 4* and *5*).

Comparing the trend of correct answers for males and females, it is clear that the gap is present in almost all ability levels, from medium-lower to higher. This is also due to the fact that there were fewer girls between top performers and that the highest decile of females displays lower results than that of the males.

Furthermore, the percentage of females choosing distractors C and D, related to didactic contract, exceeds 40% for (respectively) medium-high and medium-low ability levels, as opposed to males, for whom the percentage of these two distractors remains constantly under 40%.

We also observe a different trend of males and females in choosing options C and D. Females belonging to lower ability levels were more attracted by option D, while in higher ability levels the two options switched and the preferred option became C. On the other hand, the male distractor plot reveals no such significant difference in trend for options C and D but (only for medium-high ability levels) we can observe that distractor C proved more attractive than D.

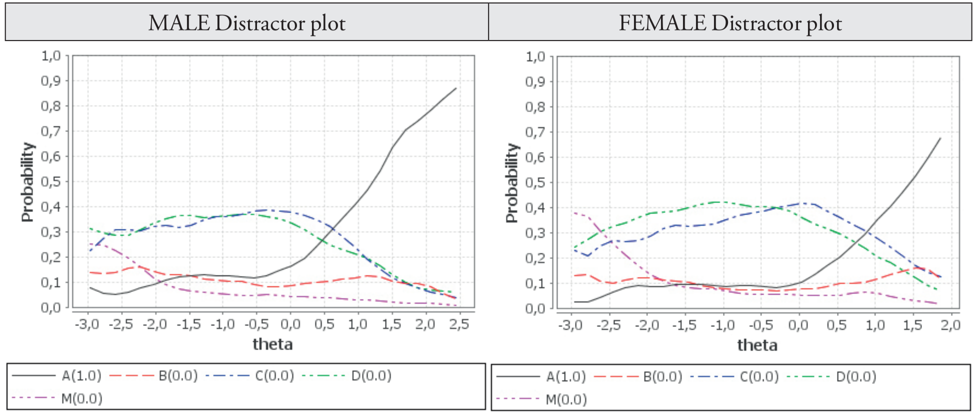

Figure 4. – Distractor plot of males and females for task D6, Mathematics INVALSI test 2013, Grade 10 students. Denser, darker lines represent female performances.

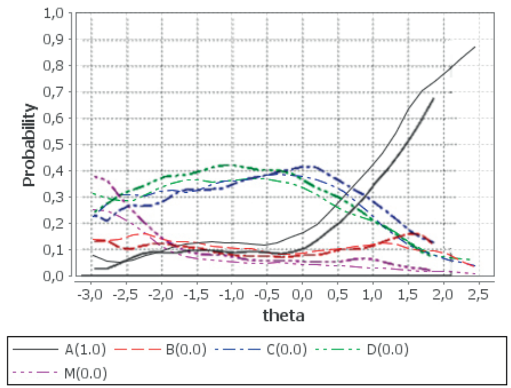

Figure 5. – Comparison of male and female distractor plots for task D6, Mathematics INVALSI test of 2013, Grade 10 students. Denser, darker lines represent female performances.

5.2. *Analysis of Grade 05 INVALSI test*

The other task investigated in this research is the following one (*Fig. 6*), administered in the INVALSI mathematics test of. 2015 for Grade 5 students. The task was administered to 562.047 fifth-year students of Primary School and the sample consisted of 22,030 students, chosen according to criteria of representativeness (INVALSI & SNV, 2015).

D7. **Francesca prepares her cat two meals a day, using cat food.**

Using one tin of cat food, Francesca prepares 3 meals for her cat.

Francesca has bought 8 tins of cat food. How many days of meals can she prepare for the cat?

A. ☐ 24

B. ☐ 16

C. ☐ 8

D. ☐ 12

Figure 6. –Task D5, Mathematics INVALSI test of 2015, Grade 5 students.

The following table reveals the percentage of correct and missing answers, and the percentage for each wrong option choice (*Tab. 3*).

Table 3. – The results of the task D5, Mathematics INVALSI test of 2015, Grade 05 students.

	Number of students	Percentage of students
A	10058	46%
B	2407	11%
C	2969	13%
D (correct)	**6363**	**29%**
M (missing)	233	1%

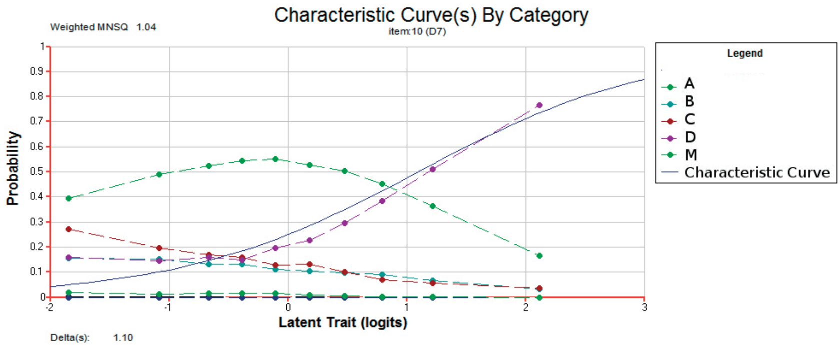

Figure 7. – Distractor plot for task D5, Mathematics INVALSI test of 2015, Grade 5 students.

The correct answer is D and only 29% of students answered this correctly. Almost all the students answered the question, indeed only 1% of answers are missing. This means that students were confident of their answers but almost half of them were attracted by option A.

The distractor plot (*Fig. 7*) confirms that the majority of students chose Option A, in which the answer is the result of multiplication of the number explicitly present in the text. The behaviour of students is related to some categories of didactic contract and in particular, it seems analogous to behaviours detected in situations related to the effect of the didactic contract known as the «Captain's Age effect» (Baruck, 1985; Verschaffel, Greer, & de Corte, 2000). The distractor plot reveals that this option is attractive for students of all ability levels but particularly for medium ability levels, and this is a typical feature of didactic contract as already highlighted in previous research studies (Ferretti *et al.*, 2018).

Once more, the connection between didactic contract and gender gap is confirmed, and the difference between the percentage of correct answers from males and females is significant (*Tab. 4*).

Table 4. – Results divided by gender of task D5, Mathematics INVALSI test of 2015, Grade 5 students.

	All students	Females	Males
A	46%	48%	44%
B	11%	11%	11%
C	13%	13%	13%
D (correct)	29%	26%	31%
M (missing)	1%	1%	1%
Gender Gap Index = 17%			

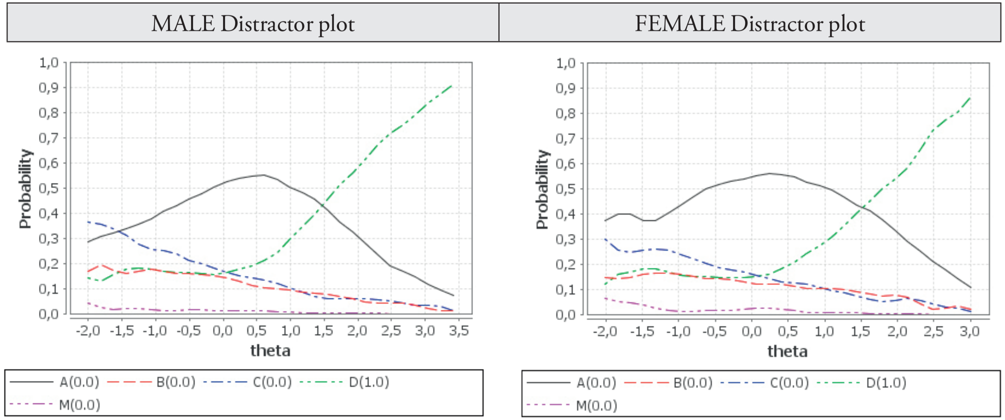

Figure 8. – Distractor plots of males and females for task D5, Mathematics INVALSI test of 2015, Grade 5 students. Denser and darker lines represent female performances.

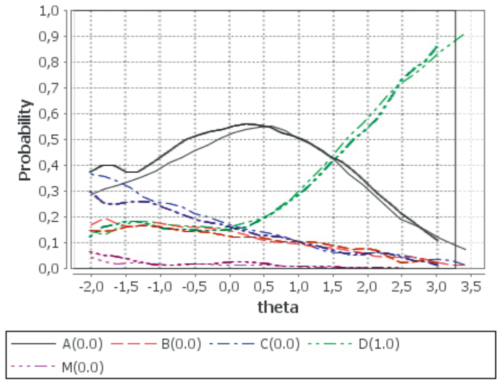

Figure 9. – Comparison of male and female distractor plots for task D5, Mathematics INVALSI test of 2015, Grade 5 students. Denser and darker lines represent female performances.

31% of boys chose the correct answer, as opposed to 26% of females. The Gender Gap Index reveals a gap in favour of boys and, even though this gap is smaller than that of the first item, it is still relatively large.

Moreover, we can observe that almost the entire gap regarding correct answers is due to option A, i.e. the answer directly linked to the effect of didactic contract. This fact confirms that girls are more influenced by didactic contract than boys and we can also observe whether this difference is present in particular ability levels by comparing the distractor plots below (*Figs. 8* and *9*).

In this item, we can observe that the trend for correct answers is almost the same for both males and females, and the difference observed in the performances is because there are fewer girls reaching the highest ability levels than boys. Furthermore, we can observe an interesting difference in the trend of option A: for lower ability levels, this option resulted more attractive to females and this means that, in this item, didactic contract influences gender gap especially for lower ability levels.

6. Conclusions and further directions

The objective of the research was to investigate the existence and characteristics of the connection between didactic contract and gender gap.

As we have seen, some evidence from the gender gap studies (Bell & Norwood, 2007; Leder & Forgasz, 2008; Matthews *et al.*, 2009; OECD, 2015; Cargnelutti *et al.*, 2016; Giberti *et al.*, 2016) and an in-depth analysis of the dynamics of classroom situations in terms of didactic contract seem to indicate that females are more predisposed to the establishment of the didactic contract.

In order to confirm the research hypotheses, we carried out a statistical analysis on some specific questions belonging to standardized mathematics INVALSI tests. In the two items selected, students' answers resulted as connected to phenomena of didactic contract (Brousseau, 1997; D'Amore, 2008); next, the statistical tools based on Rasch analysis allowed us to follow different investigation paths.

In both questions we identified a significant gender gap in favour of males; the difference is particularly due to the fact that girls are more attracted to wrong options related to didactic contract.

Then, we studied these differences in relation to the students' ability levels: we used distractor plots to analyse the trend of each answer option in the multiple choice questions and, comparing distractor plots of males and females, we also studied the gender gap in reference to each option. The analysis carried out for the two tasks highlights that the gender differences are related to the trend of the incorrect answers linked to didactic contract, and the ability levels more attracted by these options are not always the same for males and females.

Furthermore, the two questions analysed reveal different didactic phenomena, despite both referring to the didactic contract. This opens up numerous investigative tracks; in further research it will be possible to analyse other situations related to the didactic contract in order to understand better this connection, which might turn out to be one of the factors influencing gender differences in math. More generally, this kind of item-level analysis will be used to investigate other phenomena that can be interpreted with constructs of mathematics education, searching for further evidence and causes of gender gap.

A better understanding of factors influencing gender gap and features of this phenomenon can certainly offer ideas for planning and implementing system-level interventions and classroom situations aimed at reducing it.

References

Alivernini, F., Manganelli, S., Lucidi, F., Di Leo, I., & Cavicchiolo, E. (2017). Studenti svantaggiati e fattori di promozione della resilienza. *Journal of Educational, Cultural and Psychological Studies*, *16*, 35-56.

Barbaranelli, C., & Natali, E. (2005). *I test psicologici. Teorie e modelli psicometrici.* Roma: Carrocci.

Bell, K. N., & Norwood, K. (2007). Gender equity intersects with mathematics and technology: Problem-solving education for changing times. In D. Sadker & E. S. Silber (Eds.), *Gender in the classroom* (pp. 225-258). Mahwah, NJ: Lawrence Erlbaum Associates.

Bolondi, G., Cascella, C., & Giberti, C. (2017). Highlights on gender gap from Italian standardized assessment in mathematics. In J. Novotnà & H. Moravà (Eds.), *Diversity in Mathematics Education.* Proceedings of the *International Symposium Elementary Maths Teaching – SEMT 17.* Prague: Universita Karlova Press.

Bolondi, G., Cascella, C., & Giberti, C. (2018). How much item formulations affect the probability of a correct answer? An experimental study. In J. Bloggs, (Ed.), *Mathematics in the Classroom. Proceedings of the British Society of Research in Learning Mathematics*, *38*(2).

Bordeaux, I. R. E. M. (1978). Étude de l'influence de l'interprétation des activités didactiques sur les échecs électifs de l'enfant en mathématiques. Project de Recherche CNRS, Enseignement élémentaire des mathématiques. *Cahier de l'IREM de Bordeaux*, *1*(18), 170-181.

Brousseau, G. (1980a). Les échecs électifs dans l'enseignement des mathématiques à l'école élémentaire. *Revue de Laringologie Otologie Rinologie*, *101*(3-4), 107-131.

Brousseau, G. (1980b). L'échec et le contrat. *Recherches*, *41*, 177-182.

Brousseau, G. (1988). Le contrat didactique. Le milieu. *Recherches en Didactique des Mathématiques*, *9*(3), 309-336.

Brousseau, G. (1997). *Theory of didactical situations in mathematics*. Dortrecht: Kluwer.

Cargnelutti, E., Tomasetto, C., & Passolunghi, M. C. (2016). How is anxiety related to math performance in young students? A longitudinal study of Grade 2 to Grade 3 children. *Cognition & Emotion*, *31*, 755-764.

Cascella, C. (2017). Exploring the relationship between social roles in daily life and achievement gap between boys and girls in maths: Empirical evidences from italian primary school. In *11th annual International Technology, Education and Development Conference Proceedings.* Valencia: IATED Publisher. doi: 10.21125/inted.2017.2339

D'Amore, B. (2008). Epistemology, didactics of mathematics and teaching practices. *Mediterranean Journal of Research in Mathematics Education*, *7*(1), 1-22.

De Vleeschouwer, M., & Gueudet, G. (2011). Secondary-tertiary transition and evolutions of didactic contract: The example of duality in Linear Algebra. In M. Pytlak, T. Rowland, & E. Swoboda (Eds.), *Seventh Congress of the European Society of Research on Mathematics Education* (pp. 2113-2122). Rzeszów: University of Rzeszów.

Contini, D., Di Tommaso, M. L., & Mendolia, S. (2017). The gender gap in mathematics achievement: Evidence from Italian data. *Economics of Education Review*, *58*, 32-42.

Fennema, E., Carpenter, T. P., Jacobs, V. R., Franke, M. L., & Levi, L. W. (1998). New perspectives on gender differences in mathematics: A reprise. *Educational Researcher*, *27*(5), 19-21.

Ferretti, F. (2015). *L'effetto «età della Terra». Contratto didattico e principi regolativi dell'azione degli studenti in matematica.* Dissertation thesis (PhD in Mathematics), Alma Mater Studiorum Università di Bologna.

Ferretti, F., Giberti, C., & Lemmo, A. (2018). The Didactic contract to interpret some statistical evidence in mathematics standardized assessment tests. *EURASIA Journal of Mathematics, Science and Technology Education*, *14*(7), 2895-2906. https://doi.org/10.29333/ejmste/90988

Fryer, R. G., & Levitt, S. D. (2010). An empirical analysis of the gender gap in mathematics. *American Economic Journal: Applied Economics*, *2(2),* 210-240.

Fredricks, J. A., & Eccles, J. S. (2002). Children's competence and value beliefs from childhood through adolescence: Growth trajectories in two male-sex-typed domains. *Developmental Psychology*, *38*(4), 519.

Gallagher, A. M., De Lisi, R., Holst, P. C., McGillicuddy-De Lisi, A. V., Morely, M., & Cahalan, C. (2000). Gender differences in advanced mathematical problem solving. *Journal of Experimental Child Psychology*, *75*(3), 165-190.

Gallagher, A. M., & Kaufman, J. C. (2005). Gender differences in mathematics: Where we are and what we need to know. In A. M. Gallagher & J. C. Kaufman (Eds.), *Gender differences in mathematics* (pp. 316-332). New York: Cambridge University Press.

Giberti, C., Zivelonghi, A., & Bolondi, G. (2016). Gender differences and didactic contract: Analysis of two INVALSI tasks on powers properties. In C. Csikos, A. Rausch, & J. Szitanyi (Eds.), *Proceedings of the 40th Conference of the International Group for the Psychology of Mathematics Education* (p. 275). Szeged: IGPME.

González de San Román, A., & De La Rica, S. (2012). Gender gaps in PISA test scores: The impact of social norms and the mother's transmission of role attitudes. *Estudios de Economía Aplicada*, *34*(1).

Gould, S. L. (1996). *Strategies used by secondary school students in learning new concepts which require spatial visualization* [Unpublished doctoral dissertation]. Teachers College, Columbia University, New York.

Guiso, L., Monte, F., Sapienza, P., & Zingales, L. (2008). Culture, gender, and math. *Science – New York Then Washington*, *320*(5880), 1164.

Halpern, D. F., Beninger, A. S., & Straight, C. A. (2011). Sex differences in intelligence. *The Cambridge Handbook of Intelligence*, 253-272.

Herbert, J., & Stipek, D. (2005). The emergence of gender differences in children's perceptions of their academic competence. *Journal of Applied Developmental Psychology*, *26*(3), 276-295.

Hill, C., Corbett, C., & Rose, A. (2010). *Why so few? Women in science, technology, engineering, and mathematics*. Washington: American Association of University Women. http://www.aauw.org/learn/research/upload/whysofew.pdf

Impedovo, M., Orlandoni, A. & Paola, D. (2011). *Quaderni SNV N. 1-MAT. Guida sintetica alla lettura della prova di Matematica*. Frascati: INVALSI.

INVALSI & SNV (2013). *Rilevazioni nazionali sugli apprendimenti 2012/13. Rapporto tecnico.* Frascati: INVALSI.

INVALSI & SNV (2015). *Rilevazioni nazionali sugli apprendimenti 2014/15. Rapporto tecnico.* Frascati: INVALSI.

INVALSI & SNV (2016). *Rilevazione nazionale degli apprendimenti 2015/16. Rapporto risultati.* Frascati: INVALSI.

INVALSI & SNV (2017). *Rilevazione nazionale degli apprendimenti 2016/17. Rapporto tecnico.* Frascati: INVALSI.

Jacobs, J. E., & Bleeker, M. M. (2004). Girls' and boys' developing interests in math and science: Do parents matter?. *New Directions for Child and Adolescent Development*, *106*, 5-21.

Jacobs, J. E., & Eccles, J. S. (1992). The impact of mothers' gender-role stereotypic beliefs on mothers' and children's ability perceptions. *Journal of Personality and Social Psychology*, *63*(6), 932.

Leder, G. (1992). Mathematics and gender: Changing perspectives. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 597-622). New York: Macmillan.

Leder, G., & Forgasz, H. (2008). Mathematics education: New perspectives on gender. *ZDM – The International Journal on Mathematics Education*, *40*(4), 513-518.

Leder, G., & Lubienski, S. (2015). Large-Scale Test Data: Making the invisible visible. In A. Bishop, H. Tan, & T. N. Barkatsas (Eds.), *Diversity in mathematics education* (pp. 17-40). Basel: Springer International Publishing.

Matthews, J. S., Ponitz, C. C., & Morrison, F. J. (2009). Early gender differences in self-regulation and academic achievement. *Journal of Educational Psychology*, *101*(3), 689.

Mullis, I. V. S., Martin, M. O., Foy, P., & Hooper, M. (2016). *TIMSS 2015 International Results in Mathematics.* Boston: TIMSS & PIRLS International Study Center.

OECD (2015). *The ABC of gender equality in education: Aptitude, behaviour, confidence*. Paris: OECD Publishing.

OECD (2016a). *PISA 2015 – Results (Volume I): Excellence and equity in education*. Paris: OECD Publishing.

OECD (2016b). *ITALY – Country note – Results from PISA 2015*. Paris: OECD Publishing.

Pajares, F. (2005). Gender differences in mathematics self-efficacy beliefs. In A. M. Gallagher & J. C. Kaufman (Eds.), *Gender differences in mathematics: An integrative psychological approach* (pp. 294-315). New York: Cambridge University Press.

Rash, G. (1960). *Probabilistic models for some intelligence and attainment tests.* København: Danish Institute for Educational Research.

Riegle-Crumb, C. (2005). The cross-national context of the gender gap in math and science. *The Social Organization of Schooling*, 227-243.

Ruffing, S., Wach, F., S., Spinath, F. M., Brünken, R., & Karbach, J. (2015). Learning strategies and general cognitive ability as predictors of gender-specific academic achievement. *Frontiers in Psychology*, *6*, art. 1238.

Verschaffel, L., Greer, B., & De Corte, E. (2000). *Making sense of word problems*. Lisse: Swets & Zeitlinger Publishers.

Winkelmann, H., van den Heuvel-Panhuizen, M., & Robitzsch, A. (2008). Gender differences in the mathematics achievements of German primary school students: Results from a German large-scale study. *ZDM – The International Journal on Mathematics Education*, *40*(4), 601-616.

Riassunto

Sempre più ricerche a livello nazionale e internazionale, sono focalizzate sullo studio del gender gap rilevato nelle performance degli studenti in discipline di area tecnologica-scientifica. Al centro di forti dibattiti ideologici ed epistemologici ci sono in particolare le differenze in matematica, in termini di risultati ottenuti, tra maschi e femmine, che nella maggior parte delle nazioni risultano essere a favore degli studenti maschi. In questo contributo analizziamo alcuni quesiti tratti delle Prove INVALSI di matematica di diversi gradi scolastici in cui emergono fenomeni didattici interpretati con uno dei concetti chiave della didattica della matematica, il contratto didattico. Dall'analisi quantitativa, basata sulle percentuali di risposta e sullo studio di particolari grafici output del Modello di Rasch, è stato possibile evidenziare la presenza di marcate differenze di genere nei quesiti in oggetto che sono riconducibili agli effetti del contratto didattico. Inoltre lo studio delle caratteristiche con cui si presentano queste differenze di genere nelle situazioni considerate permette di indagare il gender gap in matematica attraverso una nuova chiave di lettura anche in relazione al livello di abilità degli studenti.

Parole chiave: Contratto didattico; Didattica della matematica; Differenze di genere; Prove standardizzate; Rasch analysis.

How to cite this Paper: Bolondi, G., Ferretti, F., & Giberti, C. (2018). Didactic contract as a key to interpreting gender differences in maths [Il contratto didattico come una chiave di lettura per interpretare le differenze di genere in matematica]. *Journal of Educational, Cultural and Psychological Studies*, *18*, 415-435. doi: http://dx.doi.org/10.7358/ecps-2018-018-bolo