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Contents

‘The case against fuzzy logic revisited’ revisited

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Contradictions like *The circle is big and not big* are sometimes judged true if the circle is borderline big. This observation has been taken to show that classical logic is an inadequate tool for describing linguistic meaning, and that one should instead adopt a logic that allows for degrees of truth, e.g., fuzzy logic (Alxatib et al. 2013). According to fuzzy logic, the truth value of a conjunction is the lowest truth value of its conjuncts. Hence, if *The circle is big* and *The circle is not big* are true to degrees 0.6 and 0.4, *The circle is big and not big* is true to degree 0.4.

Sauerland (2011) tested whether fuzzy logic captures people’s intuitions about contradictions. Participants marked a value between 0 and 100 to rate the truth value of two sentences—abbreviated as A and B—that were borderline true, as well as their negations $\neg A$ and $\neg B$, the conjunctions $A \wedge \neg B$ and $B \wedge \neg A$, and the contradictions $A \wedge \neg A$ and $B \wedge \neg B$. Table 1 shows the mean truth ratings. Sauerland argues that the results speak against fuzzy logic because $B \wedge \neg A$ was rated significantly lower than $A \wedge \neg A$, even though the lowest-truth-value rule predicts comparable ratings. However, the low rating for $B \wedge \neg A$ is surprising on any account, especially since $A \wedge \neg B$ behaved as expected. Moreover, fuzzy logic almost perfectly predicts the ratings for contradictions. The results are thus not unequivocal.

A	B	$\neg A$	$\neg B$	$A \wedge \neg A$	$B \wedge \neg B$	$A \wedge \neg B$	$B \wedge \neg A$
45	42	46	47	43	47	43	26

Table 1: Mean truth ratings (Sauerland 2011)

To obtain more decisive evidence, we expanded on Sauerland’s paradigm to test a much wider range of borderline and non-borderline situations instead of just one. 288 participants on MTurk each saw a sentence and five displays. There were four types of sentences:

(1) a. simple: *The circle is P*
    b. negative: *The circle is not P*
    c. conjunction: *The circle is P and not Q*
    d. contradiction: *The circle is P and not P*

P was varied between *red* and *big*; *Q* between *big* and *red*. Figure 1 shows an example display. Each display randomly varied the circle’s size and redness. For the purpose of analysis, we normalised these two dimensions. Participants had to indicate whether the sentence was true or false in each display. Thus, we tested a wide range of borderline and non-borderline situations.

Assuming that the proportion of ‘true’ responses approximates fuzzy truth (Hampton 2010), we fitted S-shaped membership functions to capture the fuzzy truth values of simple and negative sentences (Zadeh 1983). These functions are plotted in Figure 2 alongside the binned mean truth values assigned to contradictions. Afterwards, we used the membership functions to determine the fuzzy truth values for conjunctions and contradictions based on the lowest-truth-value rule, and calculated how well these correlated with participants' judgements. The correlations were high for conjunctions ($r = .81$) but not for contradictions ($r = .24$). Fuzzy logic thus provides a satisfactory account of people's intuitions about conjunctions but not contradictions.

Figure 1: Example trial.

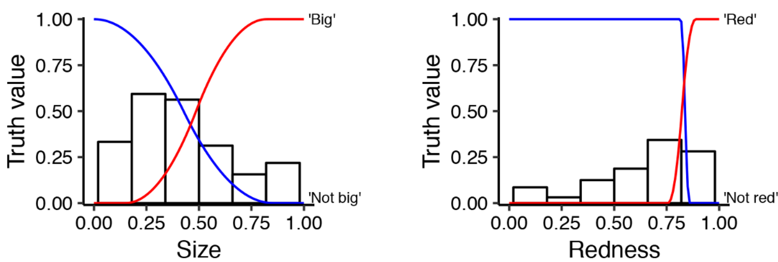

Figure 2: Binned mean truth values for *The circle is big and not big* (left) and *The circle is red and not red* (right) alongside membership functions.

References

Alxatib, Sam, Peter Pagin, and Uli Sauerland. 2013. Acceptable contradictions: pragmatics or semantics? A reply to Cobreros et al. *Journal of Philosophical Logic* 42:619–634.

Hampton, James A. 2010. Typicality, graded membership, and vagueness. *Cognitive Science* 31:355–384.

Sauerland, Uli. 2011. The case against fuzzy logic revisited. In *Understanding Vagueness: Logical, Philosophical and Linguistic Perspectives*, ed. Petr Cintula, Christian G. Fermüller, Lluis Godo, and Petr Hájek, 185–198. London, United Kingdom: College.

Zadeh, Lotfi. 1983. A computational approach to fuzzy quantifiers. *Computers & Mathematics with Applications* 9:149–184.

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